MANDURAH CATHOLIC COLLEGE

Test 4 – Trigonometry 2017

Section 1 Calculator Free

MATHEMATICS SPECIALIST UNIT 2
Year 11

Name: _Raft / cols._

Teacher: _____

Result CF:_____/15   Result CA:_____/ (17)
Total:_____/ (19)   _____%

Time allowed for this section
Working time for this paper: 15 min + 2̶0̶ min = _____ minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the student
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters.

Special items: nil

Important note to students
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Instructions to students

1. **ALL** questions should be attempted.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you **do not use pencil**, except in diagrams.

Question 1 (4 marks)

Given that $y = 4 - 2\cos\left(2x - \frac{\pi}{4}\right)$, find:

(a) The equation of the mean line (1 mark)

$$y = 4 \checkmark$$

(b) The amplitude (1 mark)

$$\text{amplitude} = 2 \checkmark$$

(c) The period (1 mark)

$$180° \checkmark$$

(d) The phase shift (1 mark)

$$= \frac{\pi}{4} \div 2$$

$$= \frac{\pi}{8} \text{ to the left} \checkmark$$

3 (4)

Question 2 (6 marks)

Solve each of the following equations for the specified domain:

(a) $\tan\left(\frac{x+25}{2}\right) = \sqrt{3}$ for $0° \le x \le 540°$. (3 marks)

Adj domain: $12.5° \le x \le 282.5°$ ✓ Adjusts domain

$$\frac{x+25}{2} = 60°, 240°$$ ✓ two sols & correct

$$x+25 = 120°, 480°$$

$$x = 95°, 455°$$ ✓ final sols.

(b) $\cos 2x = 1 + \sin x$ for the smallest possible value of $x > \pi$. (3 marks)

$$1 - 2\sin^2 x = 1 + \sin x$$ ✓ uses double angle identity

$$0 = \sin x + 2\sin^2 x$$

$$0 = \sin x (1 + 2\sin x)$$ ✓ factorises

$\sin x = 0$ or $1 + 2\sin x = 0$

Not needed

as $x > \pi$ $\sin x = -\frac{1}{2}$

$$x = \pi + \frac{\pi}{6}$$

$$= \frac{7\pi}{6}$$

Hence smallest value for $x > \pi$ is $\frac{7\pi}{6}$ ✓ correct answer,

4 ⑥

Question 3 (5 marks)

(a) Determine the exact value for $\sin\left(\frac{\pi}{12}\right)$ (3 marks)

$$\sin \frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \qquad \checkmark \; \frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$$

$$= \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \quad \checkmark \text{ uses compound angle identity}$$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4} \quad \checkmark \text{ simplify}$$

(b) Prove the identity $\tan x \cos x - \sin^3 x = \sin x \cos^2 x$. (2 marks)

$$LHS = \frac{\sin x}{\cos x} \cos x - \sin^3 x$$

$$= \sin x - \sin^3 x \qquad \checkmark \text{ getting to } \tan x \cos x = \sin x$$

$$= \sin x \left(1 - \sin^2 x\right)$$

$$= \sin x \cos^2 x \qquad \checkmark$$

$$= RHS \qquad \text{Q.E.D.}$$

END OF CALCULATOR FREE

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MANDURAH CATHOLIC COLLEGE

Test 4 – Trigonometry 2017

Section 2 Calculator Assumed

MATHEMATICS SPECIALIST UNIT 2
Year 11

Name: _____

Teacher: _____

Result CA:_____/

Time allowed for this section
Working time for this paper: _____minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the student
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters.

Special items: Scientific calculator, CAS calculator, 1 A4 (1 sided) page of notes.

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Question 4

(a) Prove the identity $\sin x + \sin 2x + \sin 3x = (1 + 2\cos x)\sin 2x$. (3 marks)

$LHS = \sin x + \sin 3x + \sin 2x$ ✓ rearranges

$= 2\sin(2x)\cos(x) + \sin 2x$ ✓ sum to product

$= \sin 2x(2\cos(x) + 1)$

$= (1 + 2\cos x)\sin 2x$ ✓ finishes proof

$= RHS \quad QED.$

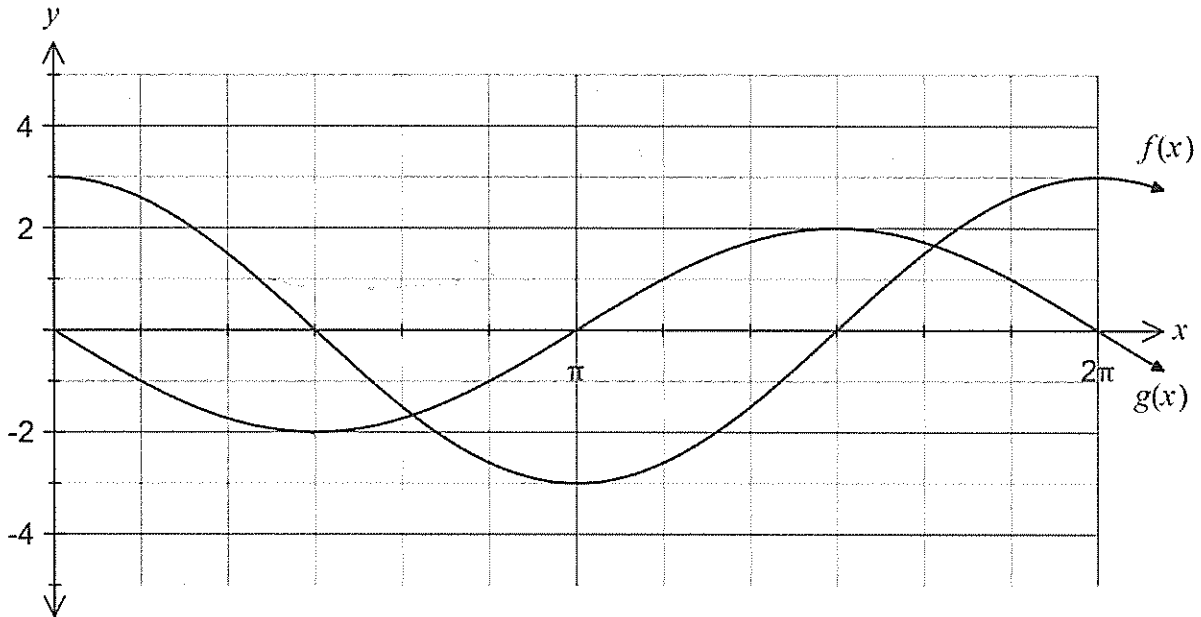(b) Prove the identity $\sin(11x)\cos(7x) - \sin(8x)\cos(4x) = \sin(3x)\cos(15x)$.

() 3 marks.

$LHS = \frac{1}{2}[\sin(18x) + \sin(4x)] - \frac{1}{2}[\sin(12x) + \sin(4x)]$ ✓ uses product to sum.

$= \frac{1}{2}(\sin(18x) - \sin(12x))$ ✓ simplifies

$= \frac{1}{2}[2\sin(3x)\cos(15x)]$

$= \sin(3x)\cos(15x)$

$= RHS \quad QED$ ✓ finishes proof,

6

Question 5 (4 marks)

The graphs of $y = f(x)$ and $y = g(x)$ are shown below for $0 \leq x \leq 2\pi$.

(a) If $f(x) = a \cos x$ and $g(x) = b \sin x$, state the values of a and b. (1 mark)

$$a = 3 \quad , \quad b = -2$$

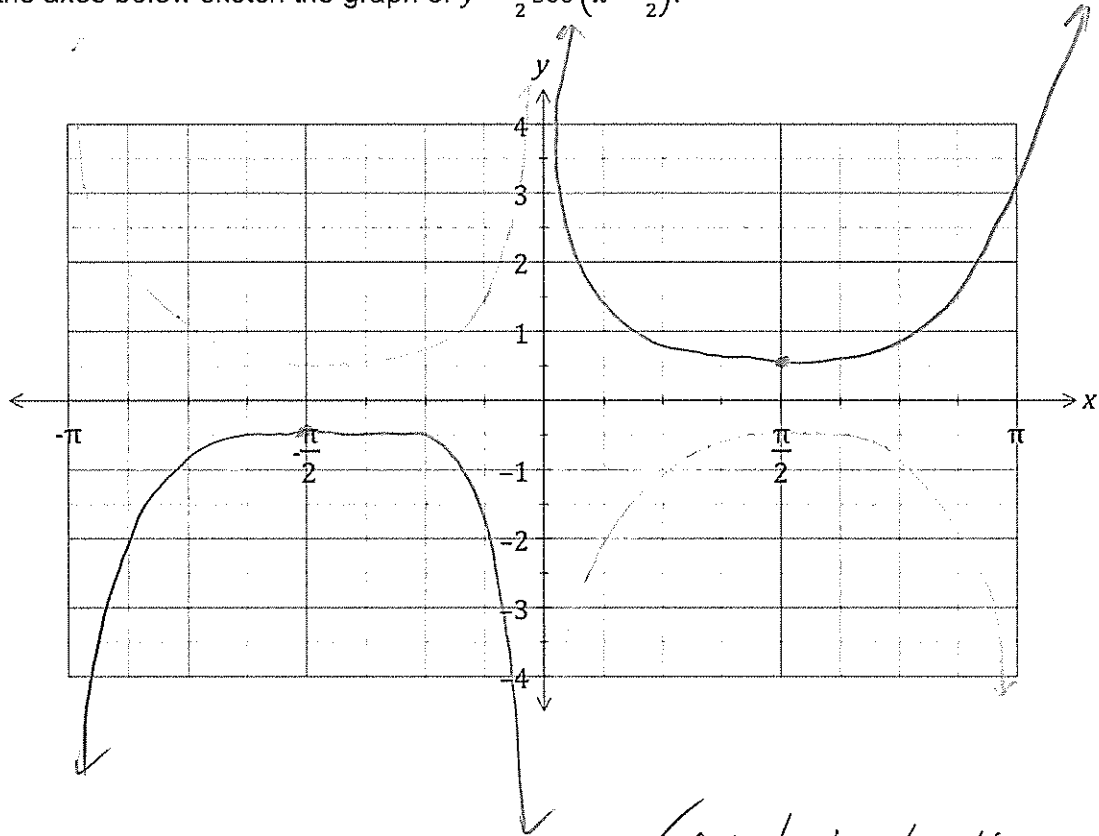(b) If $h(x) = f(x) + g(x)$ express $h(x)$ in the form $R \cos(x + \alpha)$. (3 marks)

$h(x) = 3 \cos x - 2 \sin x$

$= \sqrt{3^2 + 2^2} \ \cos\left[x + \tan^{-1}\left(\frac{2}{3}\right)\right]$

$= \sqrt{13} \ \cos(x + 0.588)$

✓ R value

✓ $\cos(x - \alpha)$

✓ in radians.

Question 6 (3 marks)

On the axes below sketch the graph of $y = \frac{1}{2}\sec\left(x - \frac{\pi}{2}\right)$.

✓ max/min locations

✓ vertical asymptotes

✓ smooth / correct graph.

③

Question 7 (6 marks)

For the function $f(x) = 2\sin^3 x + 5\sin^2 x + \sin x - 2$:

(a) Given that a solution of $f(x)$ is $\sin x = \frac{1}{2}$, factorise $f(x)$ showing all working. (4 marks)

Hence $(2\sin x - 1)$ is a factor ✓ recognises factor.

$f(x) = (2\sin x - 1)(a\sin^2 x + b\sin x + c)$

let $\sin x = y$ ✓ uses factor theorem.

$f(y) = (2y - 1)(ay^2 + by + c)$ ∴ $a = 1$, $c = 2$

$= (2y - 1)(y^2 + 3y + 2)$ $2b - 1 = 5$ ✓ correct coefficients

$= (2y - 1)(y + 1)(y + 2)$ ✓ factorises quadratic $b = 3$ for quadratic

Hence $f(x) = (2\sin x - 1)(\sin x + 1)(\sin x + 2)$

(b) Solve the equation for $f(x) = 0$ for all values of x in degrees. (2 marks)

$\sin x = \frac{1}{2}$, $\sin x = -1$, $\sin x = -2$

$x = (-1)^n(-90°) + 360°n$, $(-1)^n(30°) + 360°n$ $n \in \mathbb{Z}$ ✓ Both solns
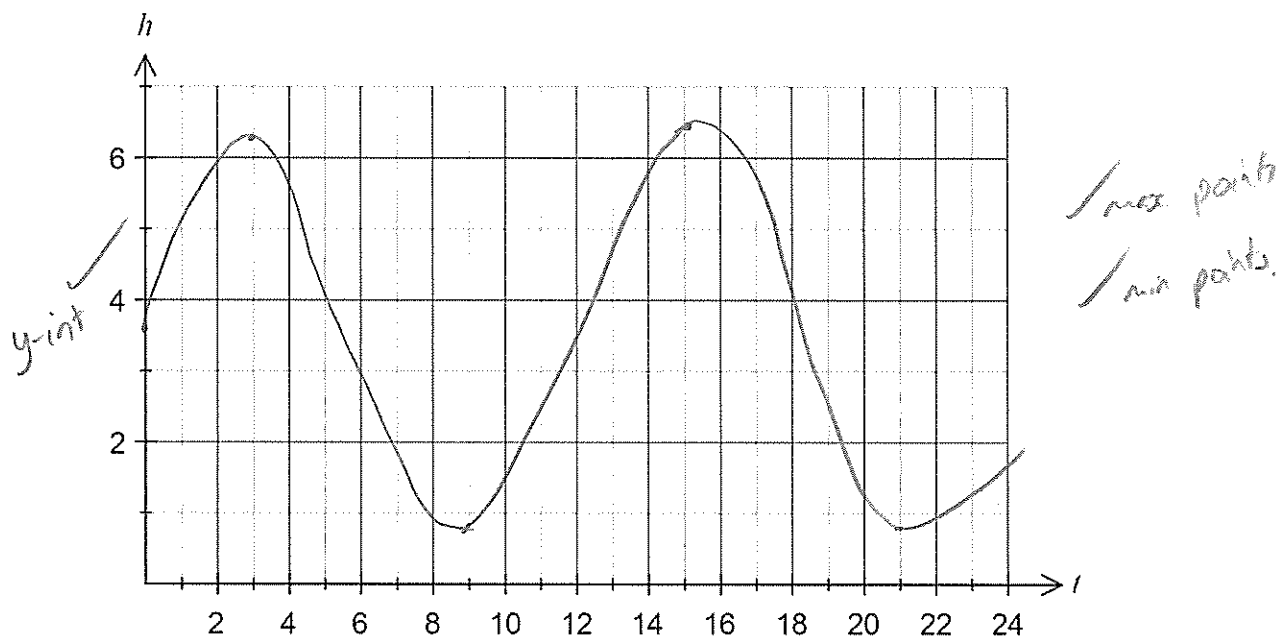
END OF ASSESSMENT

Question 8 (5 marks)

The clearance, h metres, under a bridge spanning a river estuary varies with the time since midnight, t hours, and is given by $h = 3.6 + 2.7\sin\left(\dfrac{\pi t}{6}\right)$.

(a) Sketch the graph of the clearance against time on the axes below. (3 marks)

(a) Determine the percentage of any 24-hour period during which the clearance under the bridge is no more than two metres. (2 marks)

$h \leq 2 \Rightarrow 7.21 \leq t \leq 10.79$ ✓ identify range(s)

$= \dfrac{10.79 - 7.21}{12} \times 100 = 29.8\%$ ✓ calc %